Remarkable features of multiplier distributions in turbulence

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Multiplicative cascades provide a general framework to describe intermittency properties of turbulent fluctuations [1–3]. The locally (spatially) averaged energy dissipation rate ε_r is a fluctuating quantity of special interest. The issue consists in relating fluctuations of ε_r at different scales r_1 and $r_2 \leq r_1$. A common approach is based on the introduction of a random multiplier W_{r_1,r_2}, connecting events $\varepsilon_{r_1}(x_1)$ and $\varepsilon_{r_2}(x_2)$ at location x_1 and x_2:

$$\varepsilon_{r_2}(x_2) = W_{r_1,r_2}\ \varepsilon_{r_1}(x_1). \tag{1}$$

The statistics of the multiplier W_{r_1,r_2} (or $\log W_{r_1,r_2}$) characterize the cascade properties of ε_r. In the present study, we concentrate on the (arbitrary) case of centered events, i.e. $x_1 = x_2 = x$ in eq.(1). We describe the statistics of $\log W_{r_1,r_2}$ within the framework of a new parametric model.

We analyse hot-wire velocity measurements in a turbulent jet ($R_\lambda \sim 600$) [4]. The integral scale, estimated as the velocity correlation length, is $L \simeq 2000\eta$ where $\eta \simeq 120\mu$m denotes the Kolmogorov scale. Data have been resampled by use of a local Taylor's hypothesis. The one-dimensional surrogate of energy dissipation is used. The multiplier is defined by

$$0 \leq W_{r_1,r_2} = \frac{\varepsilon_{r_2}(x)}{\varepsilon_{r_1}(x)} \leq \frac{r_1}{r_2} \quad \text{with} \quad \varepsilon_r(x) = \frac{1}{r}\int_{x-r/2}^{x+r/2} \varepsilon(x')dx'. \tag{2}$$

For any pair of scales (r_1, r_2), satisfying $\eta \lesssim r_2 \leq r_1 \lesssim L$, we propose to describe the statistics of $\log W_{r_1,r_2}$ by considering $W = \alpha\beta^w$, where

$$w = \frac{\log W_{r_1,r_2} - \log \alpha_{r_1,r_2}}{\log \beta_{r_1,r_2}} \geq 0 \tag{3}$$

has a probability density function (pdf) of the form

$$\mathrm{pdf}(w) \propto \frac{\lambda^w_{r_1,r_2}}{\Gamma(w+1)}. \tag{4}$$

This pdf may be viewed as a continuous generalization of the standard Poisson law of parameter λ used in [5]. Our choice is arbitrary but suitably captures the main features of the experimental pdfs (displayed in Fig. 1a): the Poisson distribution of parameter λ accounts for the dissymmetry, the large-amplitude cut-off of the pdf is given by $\log\alpha$ and β is an amplification factor. The parameters are estimated as follows

$$\widehat{\log\alpha} = \max(\log W_{r_1,r_2}) = \log(\max W_{r_1,r_2}) \tag{5}$$

$$\widehat{\beta} = \exp\left\{\frac{\ll \log W - \widehat{\log\alpha} \gg}{\langle \log W - \widehat{\log\alpha}\rangle}\right\} = \exp\left\{\frac{\ll \log(\frac{W}{\max W}) \gg}{\langle \log(\frac{W}{\max W})\rangle}\right\} \tag{6}$$

$$\widehat{\lambda} = \frac{\langle \log W - \widehat{\log\alpha}\rangle}{\widehat{\log\beta}} = \frac{\langle \log(\frac{W}{\max W})\rangle^2}{\ll \log(\frac{W}{\max W}) \gg}, \tag{7}$$

where $\langle\,.\,\rangle$ and $\ll\,.\,\gg$ denote respectively the estimated mean and variance. This procedure has been applied for $500\eta \le r_1 \le 6000\eta$ and $\eta \le r_2 \le r_1$. The agreement between the experimental pdfs and the model is very satisfactory (see Fig. 1a). Some discrepancies are observed only as r_2 approaches the limiting scales η and r_1. Furthermore, α, β and λ display remarkably simple laws (see Fig. 1b-d):

$$\widehat{\log\alpha}_{r_1,r_2} \simeq -0.7\log\frac{r_2}{r_1}, \qquad \text{for all } r_2/r_1, \tag{8}$$

$$\hat{\beta}_{r_1,r_2} \simeq 0.12\log\frac{r_2}{r_1} + 0.92, \qquad \text{for all } r_2/r_1, \tag{9}$$

$$\hat{\lambda}_{r_1,r_2} \simeq -2\log\frac{r_2}{r_1} + 5.4 \qquad \text{for } 10\eta \lesssim r_2 \lesssim r_1/3. \tag{10}$$

These results clearly show that $\log r_2/r_1$ appears as the relevant quantity to describe the scale dependence of $\widehat{\log\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$. This is not a trivial effect since the estimation procedure uses no *a priori* assumption on the functional dependencies of the parameters with r_1 and r_2. We have noticed that these laws are valid in a wider range of scales r_2 than that where the agreement between experimental and theoretical pdfs was observed.

We now intend to compare these experimental observations with the predictions of the Log-Poisson / She-Lévêque model (LP/SL) [5, 6], which has recently received much interest. In this model, the variables $\log W_{r_1,r_2}$ are independent and satisfy eq.(3) where w is a (discrete) Poisson variable of parameter λ^{LP}.

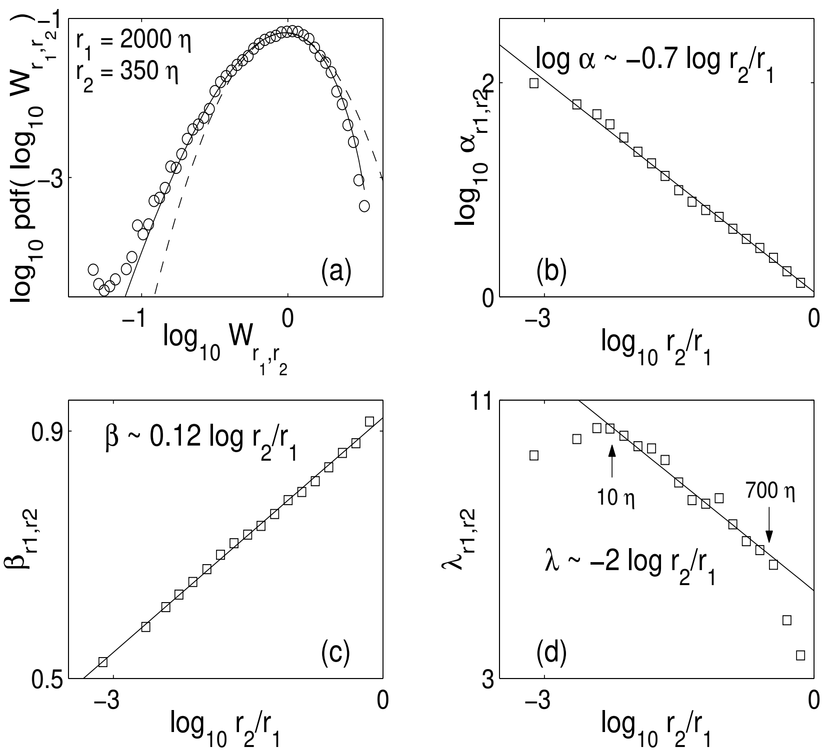

Figure 1: (a): Experimental pdf is well described by (3) and (4) (solid line). Note that the Log-Normal pdf (dashed line) is ruled out. (b-d): Evolution of the parameters with scale r_2. Solid line indicates a characteristic behavior. Remark that $\lim_{r_2 \to r_1} \alpha_{r_1,r_2} = 1$ and $\lim_{r_2 \to r_1} \beta_{r_1,r_2} \simeq 1$; this is consistent with $W_{r_1,r_1} = 1$.

Moreover, α^{LP}, β^{LP} and λ^{LP} satisfy for all r_2/r_1

$$\log \alpha^{LP} = -\frac{2}{3} \log \frac{r_2}{r_1}, \tag{11}$$

$$\beta^{LP} = \frac{2}{3}, \tag{12}$$

$$\lambda^{LP} = -2 \log \frac{r_2}{r_1}, \tag{13}$$

where the values of the constants are derived from physical arguments in [6]. The SP/SL process is multiplicative and log-infinitely divisible (see [5–7]). It yields power-law scaling for the moments of W_{r_1,r_2}:

$$\log \mathbb{E}\left[W_{r_1,r_2}^q\right] = q \log \alpha^{LP} - \lambda^{LP} \left(1 - \left(\beta^{LP}\right)^q\right) = \log \frac{r_2}{r_1} \cdot H(q), \tag{14}$$

with $H(q) = -2/3q + 2(1-(2/3)^q)$. The key feature of eq.(14) lies in the separation of the $\log r_2/r_1$ and q dependencies.

The agreement between this model and our estimations is excellent for $\widehat{\log\alpha}$ and $\hat{\lambda}$. Our experimental results support both the $\log r_2/r_1$ dependence and the numerical values of the constants. For $\hat{\beta}$ a significant discrepancy is observed. Our results show a clear $\log r_2/r_1$ behaviour while the LP/SL model involves a constant value $\beta^{LP} = 2/3$. We observe that $0.5 \lesssim \hat{\beta} \lesssim 1$ but the specific value $2/3$ cannot be singled out. The (r_1, r_2)-dependence of β is not consistent with the separation of the variables $\log r_2/r_1$ and q in $\log \mathbb{E}\left[W^q_{r_1,r_2}\right]$. Therefore, it *excludes* the possibility of a log-infinitely divisible cascade and *a fortiori* the power-law scaling hypothesis [7, 8].

We experimentally observe that the statistics of the multipliers W_{r_1,r_2} for the energy dissipation ε_r are suitably described by a continuous generalisation of the usual Log-Poisson model. The estimated parameters exhibit very simple behaviours as affine functions of $\log r_2/r_1$. This supports one of the main assumptions of the LP/SL model, i.e. that $\log(\max W_{r_1,r_2})$ behaves as $\log r_2/r_1$. Importantly, it also implies that log-infinite divisibility with independent multipliers, usually involved in the modeling of the energy dissipation cascade, should be dropped out.

References

[1] Frisch, U.: 1995, *Turbulence. The legacy of A. Kolmogorov.* Cambridge, UK: Cambridge University Press.

[2] Chhabra, A. B. and Sreenivasan, K. R.: 1992, *Phys. Rev. Lett.* **68**(18), 2762.

[3] Pedrizzetti, G., Novikov, E. A. and Praskovsky, A. A.: 1996, *Phys. Rev. E* **53**(1), 475.

[4] Baudet, C., Michel, O. and Williams, W. J.: 1999, *Physica D* **128**, 1.

[5] Dubrulle, B.: 1994, *Phys. Rev. Lett.* **73**, 959 – She, Z. S. and Waymire, E. C.: 1995, *Phys. Rev. Lett.* **74**(2), 262.

[6] She, Z. S. and Lévêque, E.: 1994, *Phys. Rev. Lett.* **72**(3), 336.

[7] Castaing, B., Gagne, Y. and Hopfinger, E.: 1990, *Physica D* **46**, 177.

[8] Chainais, P., Abry, P. and Pinton, J.-F.: 1999, *Phys. Fluids* **11**(11), 3524.